

# IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

## Patent Application

Inventors: Kirk K. CHANG, Daniel R. JESKE, Kiran M. REGE and  
Yung-Terng WANG

Docket No.: 2925-253P

Lucent Case No.: 116711/CHANG 5-3-17-19

Title: LIKELIHOOD-BASED GEOLOCATION PREDICTION ALGORITHMS  
FOR CDMA SYSTEMS USING PILOT STRENGTH MEASUREMENTS

ASSISTANT COMMISSIONER FOR PATENTS  
WASHINGTON, D.C. 20231

Date: July 26, 1999

Sir:

Enclosed are the following papers relating to the above-named application for patent:

Specification  
Formal Drawings (eight (8) sheets)  
Assignment with Cover Sheet  
Declaration and Power of Attorney

The fee has been calculated as shown below:

CLAIMS AS FILED				
	NO. FILED	NO. EXTRA	RATE	CALCULATIONS
Total Claims	20- 20 =	0	x \$18. =	0
Independent Claims	2-3	0	x \$78. =	0
Multiple Dependent Claim(s), if applicable			x \$260. =	
BASIC FEE				\$760.00
			<b>TOTAL FEE</b>	<b>\$760..00</b>

1c588 U.S. PTO  
09/359648  
07/26/99

07/26/99  
1c588 U.S. PTO

09359648 072699

Please file the application and charge **Lucent Technologies Deposit Account No. 12-2325** the amount of **\$760.00** to cover the filing fee. Triplicate copies of this letter are enclosed. In the event of non-payment or improper payment of a required fee, the Commissioner is authorized to charge or to credit **Deposit Account No. 12-2325** as required to correct the error.

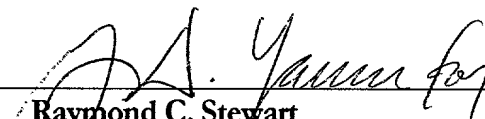
Please address all correspondence to:

BIRCH, STEWART, KOLASCH & BIRCH, LLP  
P.O. Box 747  
Falls Church, Virginia 22040-0747

Telephone inquiries may be directed to the undersigned representative at (703) 205-8000.

Respectfully submitted,

BIRCH, STEWART, KOLASCH & BIRCH, LLP

By:   
Raymond C. Stewart  
Reg. No. 21,066

Attorney for Applicant

RCS/WLG/mpe

**LIKELIHOOD-BASED GEOLOCATION PREDICTION ALGORITHMS  
FOR CDMA SYSTEMS USING PILOT STRENGTH MEASUREMENTS**Cross Reference To Related Applications

This application is related to U.S. Serial No. 09/139,107 entitled "Pattern Recognition-Based Geolocation", filed in the names of T.C. Chiang et al on August 26, 1998; U.S. Serial No. 09/294,997 entitled "A Bayesian-Update Based Location Prediction Method For CDMA Systems", filed in the names of K.K. Chang et al on April 20, 1999; and U.S. Serial No. \_\_\_\_\_, (Lucent file #CHANG 4-2-16) entitled "Geolocation Estimation Method For CDMA Terminals Based On Pilot Strength Measurements", filed in the names of K.K. CHANG et al on May 28, 1999. These related applications are assigned to the assignee of the present invention and are meant to be incorporated herein by reference.

Background Of The InventionField of the Invention

The present invention relates to a method of locating a mobile telephone unit within a cellular service area, and more particularly to a method of predicting the location of a CDMA mobile unit based upon the probability of its being at a particular location of the service area using an algorithm providing a likelihood estimation of the mobile unit's location in response to a sequential set of attributes observed by the mobile unit and reported back to a base station.

Description of Related Art

A cellular telephone system must be able to locate a mobile unit within a cellular service area under various RF propagation conditions such, for example, when an E911 call is made from the mobile unit. Conventional methods for locating a mobile unit are typically based on either a triangulation technique which requires signals from three or more base stations within a designated service area, or an angle of arrival technique which requires at least two base stations. In many areas, the number of base stations the mobile unit can detect is less than two. Furthermore, both the triangulation and angle of arrival techniques inherently suffer from inaccuracies and signal fading which result from multi-path propagation.

In the first above-noted related patent application U.S. Serial No. 09/139,107 entitled "Pattern Recognition-Based Geolocation", RF characteristics pertaining to one or more pilot signals radiated from a base station and specific to a particular location within the service area are detected by a mobile unit and transmitted back to a base station where they are matched to a known set of RF characteristics and other information obtained from making attribute information measurements at all the grid points (sub-cells) in a cellular service area and which are then stored in a database located, for example, in a base station server.

In the second above-noted related patent application U.S. Serial No. 09/294997 entitled "A Bayesian-Update Based Location Prediction Method For CDMA systems", the invention is directed to a method of estimating, by a Bayesian probability algorithm, the location of a mobile unit in the service area of a CDMA cellular telephone system using a model based approach which, among other things, simplifies the generation of a database containing a pilot signal visibility probabilities. This eliminates the need for repeated attribute measurements at all of the grid points in the service area.

In the third above-noted related patent application U.S. Serial No. \_\_\_\_\_, entitled "Geolocation Estimation Method For CDMA Terminals Based On Pilot Strength Measurements", the invention is directed to a method of estimating the location of a mobile unit in the service area of a CDMA cellular telephone system also using a model based approach, but which now eliminates the need for a stored database containing pilot signal visibility probabilities for all of the grid points or sub-cells in the cellular service area. The estimation procedure is based entirely on analytical results involving one or more key approximations derived, for example, from an integrated model of the wireless communications system, its RF environment, and attribute measurement.

#### Summary

The subject invention is directed to predicting the location of a mobile wireless communication unit in the service area of a CDMA communications system utilizing two likelihood functions that define maximum likelihood estimators of the mobile unit's location, based on attribute measurements, such as but not limited to pilot signal strength, being made at the location of the mobile unit and reported back to a base station. One of the likelihood functions comprises a frequentist likelihood function and the other comprises a

Bayesian-modified likelihood function. The likelihood functions are based on the assumption that there is an RF model which provides the probability a mobile unit is able to detect one or more attributes associated with an arbitrary base station, given it is located at an arbitrary location within the service area. The frequentist likelihood assumes the RF model provides exact visibility probabilities. In contrast, the Bayesian-modified likelihood assumes the RF model only provides reasonable approximations to the true visibility probabilities, and uses the approximations to construct a Bayesian prior distribution for the true values. Each of the likelihoods can be used in an iterative fashion to produce a maximum likelihood estimator for the location of the mobile unit by determining the coordinates within the service area which maximize the respective likelihood function. Alternatively, or in addition to, each of the likelihoods can be incorporated into a sequential Bayesian procedure which outputs a posterior distribution for the location of the mobile unit.

#### Brief Description of the Drawings

Figure 1 is illustrative of a cellular service area divided into a plurality of cells;  
Figure 2 is illustrative of the cells shown in Figure 1 being further divided into sub-cells;  
Figure 3 is illustrative of an embodiment of the subject invention; and  
Figure 4 including Figures 4A-4E comprise flow charts which are illustrative of the preferred methods of determining the locality of a mobile unit in accordance with the subject invention.

#### Detailed Description of the Invention

Referring now to the drawings and more particularly to Figure 1, the reference numeral 10 denotes a service area for a CDMA cellular telephone system partitioned into a plurality of contiguous cells  $12_1, \dots, 12_n$ . Figure 1 also depicts a plurality of base stations  $14_1, \dots, 14_n$  located within the service area 10. Also, the service area 10 includes at least one mobile switching center (MSC) 16. Typically each of the base stations  $14_1, \dots, 14_n$  has a sectorized antenna with a distinct pilot signal channel associated with each sector. Three sectorized antennas are most common. Each sector of the antenna serves a corresponding sector of the associated cell. In Figure 1, all of the base stations

14<sub>1</sub>, ..., 14<sub>n</sub> have three sectors each. The three sectors associated with base stations 14<sub>1</sub>, for example, are denoted by the symbols 15<sub>1,1</sub>, 15<sub>1,2</sub> and 15<sub>1,3</sub>, respectively. A mobile unit 20 is shown in sector 15<sub>2,1</sub>.

Figure 2 is illustrative of the cells 12<sub>1</sub>, ..., 12<sub>n</sub> being further divided into sub-cells 18 and which are represented by a grid formed by rectilinear grid lines 20 and 22. The reference numbers 1, 2, 3 ... 6 of Figure 2 represent individual sub-cells 18<sub>1</sub>, ..., 18<sub>6</sub>, respectively.

Turning attention now to Figure 3, shown thereat is a diagram broadly illustrative of the system architecture for determining the location of a mobile unit 20 within the service area 10 in accordance with the subject invention. The MSC 16 operates in conjunction with the plurality of base stations 14<sub>1</sub>, ..., 14<sub>n</sub> and connects to the local telephone system, not shown. A server 22 including digital computer apparatus 23 and memory 24, for storing computation procedures, model parameters and system data, are typically located at the site of the MSC 16 for purposes which will now be explained.

In the invention described in the first referenced related application Serial No. 09/139,107 entitled "Pattern Recognition-Based Geolocation," each sub-cell 18<sub>1</sub>, ..., 18<sub>n</sub> of the service area 10 is identified by a set of observable characteristics which are referred to as attributes. Examples of attributes are pilot signal strengths ( $E_c/I_o$ ), phase-offsets, angles of arrival, and pilot round trip delays. The invention of Serial No. 09/139,107 includes a database which contains attribute information which differentiates one sub-cell 18 from another and is generated by making a repeated and exhaustive survey which involves taking repeated measurements at all the sub-cells 18<sub>1</sub>, ..., 18<sub>n</sub> (Figure 2) of the service area 10.

During the operation phase, after the database has been set up and the location service has been deployed, the mobile unit 20 detects and measures attribute values from its actual location in sub-cell 18<sub>i</sub> and reports them via a message, e.g., a pilot signal strength measurement message (PSMM), to the base station(s) 14<sub>1</sub>, ..., 14<sub>n</sub> (Figure 3), which can be one or more of the base stations with which it is in communication. The base station(s) forward their respective reported measurements to the geolocation server

22. The digital computer apparatus 23 associated with the server 22 statistically compares the measured values with the known attribute values stored in the database (memory) 24 of all the sub-cells 18 in the service area 10. The sub-cell 18<sub>i</sub> whose attribute values as stored in the database provide the best match for the measurements reported by the mobile unit 20 is considered to be the best estimate of the mobile unit's location.

In the second referenced related application, Serial No. 09/294,997, entitled "A Bayesian-Update Based Location Prediction Method For CDMA systems", a database is also used to assist the process of location estimation. However, in contrast to the first referenced patent application, i.e. Serial No. 09/139,107, it uses a model based approach to generate a database containing pilot visibility probabilities for different sub-cells 18 in the service area 10. The model-based approach requires that a limited number of pilot strength measurements be carried out along a few representative routes in the service area 10. These measurements are then used to identify the parameters of the model that characterizes the service area and its RF environment. Once these parameters are identified, simulations are then carried out to populate the database containing the pilot visibility probabilities, which are used in the computation of the location distribution of a mobile unit requesting location service. An iterative procedure based on a Bayesian probability computation is then used to obtain improved estimates of the mobile unit's location in response to multiple sets of attribute measurements being reported by the mobile unit 20. The model-based approach eliminates the need to carry out extensive measurements required by the first named invention, Serial No. 09/139,107.

In the third referenced related patent application Serial No. \_\_\_\_\_, entitled "Geolocation Estimation Method For CDMA Terminals Based On Pilot Strength Measurements", the model-based approach embodied in Serial No. 09/294,997, "A Bayesian-Update Based Location Prediction Method...." to characterize the RF environment is used, as is the iterative procedure for computing the Bayesian posterior distribution for the location of the mobile. However, the database containing pilot visibility probabilities is replaced by analytical formulas that can be evaluated in real

time. The evaluation procedures are compact and can typically be evaluated in the digital computer apparatus 23 shown in Figure 3.

Considering the present invention, the analytic formulation for the pilot visibility probabilities taught in the third referenced application Serial No. \_\_\_\_\_, "Geolocation Estimation Method For CDMA terminals Based On Pilot Strength Measurements", now serve as the starting point for the derivation of two likelihood functions, hereafter referred to as the frequentist and Bayes-modified likelihood functions, respectively. Each of the likelihood functions is derived based on the assumptions and mathematical formulations described in attached Appendix A. In as much as the likelihood functions depend on the analytic evaluation of the pilot visibility probabilities, attached Appendix B provides a self-contained development of the relevant details of these formulas. Each likelihood function is a function of  $(x, y)$ , an arbitrary location of the mobile unit 20. Accordingly, each likelihood function could be used to obtain a maximum likelihood (ML) estimator of the location of the mobile unit 20 by finding the  $(x, y)$  which maximizes the value of the respective likelihood function. An iterative technique for sequentially updating each ML estimator with additional pilot signal strength measurements can be utilized. Each of the two likelihood functions can also be incorporated into a sequential Bayesian procedure, which outputs a posterior distribution for the location of the mobile.

The Bayes-modified likelihood function, whether it is used in the context of ML estimation or a sequential Bayesian procedure, is a substantial deviation from the inventions disclosed in the second and third above referenced related applications in the following way. Both of these previously disclosed inventions use an RF model to estimate pilot visibility probabilities, the former via simulation techniques, the latter via analytical formula evaluation, and implicitly assume that the model holds precisely. The present invention, however, uses the same RF model only to determine the means of beta distributions that are used as Bayesian priors for the true (unknown) pilot visibility probabilities. A beta distribution is completely determined once its mean and variance have been specified. Subject to the fixed mean values, each of the beta distributions is fully specified by maximizing their variances. Maximizing the variance of the beta priors, subject to the specified mean values, is consistent with a non-informative (vague) prior specification.



A summary of the derivations which appear in Appendix A will now be given as prefatory remarks to the description of the overall mobile unit 20 location prediction process depicted in Figure 4. In the example shown in Figures 1 and 2, if the primary base station for the mobile unit 20 is 14<sub>2</sub>, the region designated as 17 in Figures 1 and 2 is the set of feasible locations of the mobile unit 20, and will hereafter be referred to as the region *A*.  
5 Along with the region *A*, the set of all pilots *K* which are likely to be visible at some of the grid points 18 in the set *A* is defined.

For each  $(x, y) \in A$ , let  $\theta_{ij}(x, y)$  denote the true probability that the mobile unit 20 is able to see the pilot in sector *j* of base station *i* when it is located at  $(x, y)$ . From hereon,  
10 the notation *ij* will be used to exclusively reference pilots from the set *K*. Let  $\tilde{\theta}_{ij}(x, y)$  denote an approximation of  $\theta_{ij}(x, y)$  based on an RF model described in Appendix B. For each pilot *ij*, let  $\mu_{ij}^s$  equal one or zero depending on whether the mobile unit 20 can see pilot *ij* at the *s*-th measurement epoch or not, respectively. The frequentist likelihood through the first *s* measurement epochs has the following recursive form, starting with the definition  
15  $L_{ML}^0(x, y) \equiv 1$ :

$$L_{ML}^s(x, y) \propto L_{ML}^{s-1}(x, y) \prod_{ij \in K} [\tilde{\theta}_{ij}(x, y)]^{\mu_{ij}^s} [1 - \tilde{\theta}_{ij}(x, y)]^{1 - \mu_{ij}^s}, \quad (x, y) \in A \quad (1)$$

For the Bayes-modified likelihood, the prior for  $\theta_{ij}(x, y)$  is a beta distribution with parameters:

$$\alpha_{ij}(x, y) = \begin{cases} 1 & , \text{if } \tilde{\theta}_{ij}(x, y) \leq 1/2 \\ \tilde{\theta}_{ij}(x, y) & , \text{if } \tilde{\theta}_{ij}(x, y) > 1/2 \end{cases} \quad (2)$$

$$\beta_{ij}(x, y) = \begin{cases} \frac{1 - \tilde{\theta}_{ij}(x, y)}{\tilde{\theta}_{ij}(x, y)} & , \text{if } \tilde{\theta}_{ij}(x, y) \leq 1/2 \\ 1 & , \text{if } \tilde{\theta}_{ij}(x, y) > 1/2. \end{cases} \quad (3)$$

25 The Bayes-modified likelihood function through the first *s* measurement epochs has the following recursive form starting with the definition  $L_{BML}^0(x, y) \equiv 1$  for all

$(x, y) \in A$ :

$$L_{BML}^s(x, y) \propto L_{BML}^{s-1}(x, y) \prod_{ij \in K} \frac{[n_{ij}^{s-1} + \alpha_{ij}(x, y)]^{\mu_{ij}^s} [s-1 - n_{ij}^{s-1} + \beta_{ij}(x, y)]^{1-\mu_{ij}^s}}{\alpha_{ij}(x, y) + \beta_{ij}(x, y) + s-1}, (x, y) \in A \quad (4)$$

where  $n_{ij}^{s-1} = \sum_{k=1}^{s-1} \mu_{ij}^k$  is the number of times pilot  $ij$  was visible amongst the first  $s-1$  measurement epochs.

- 5 Each of the likelihood functions (1) and (4) are functions of  $(x, y) \in A$ , an arbitrary possible location for the mobile unit 20. The ML estimator for the location of the mobile unit 20 is obtained by evaluating (1) for all  $(x, y) \in A$  and selecting the values, say  $(x_{ML}^s, y_{ML}^s)$ , which gives the largest value of (1). An updated ML estimate is produced at each measurement epoch. In a similar way, using function (4) rather than function (1)
- 10 generates a sequence of Bayes-modified ML estimates, say  $(x_{BML}^s, y_{BML}^s)$ .

Utilizing functions (1) or (4) with a Bayesian sequential procedure can generate an alternative sequence of predictions for the location of the mobile unit 20. In each case, the initial prior distribution for the location of the mobile unit 20 is assumed to be a discrete uniform distribution of the form:

$$15 \quad P_{ML}^0(x, y) = P_{BML}^0(x, y) = \frac{1}{\|A\|}, \quad (x, y) \in A \quad (5)$$

where  $\|A\|$  is the number of grid points  $18_1, \dots, 18_n$  contained within  $A$ . The posterior distribution for the location of the mobile unit 20, through  $s$  measurement epochs, based on the frequentist likelihood function (1) is, up to a constant of proportionality:

$$P_{ML}^s(x, y) \propto P_{ML}^{s-1}(x, y) \prod_{ij \in K} [\tilde{\theta}_{ij}(x, y)]^{\mu_{ij}^s} [1 - \tilde{\theta}_{ij}(x, y)]^{1-\mu_{ij}^s}, \quad (x, y) \in A. \quad (6)$$

- 20 Alternatively, the posterior distribution of the location of the mobile unit 20, through  $s$  measurement epochs, based on the Bayes-modified likelihood function (4) is, up to a constant of proportionality:

$$P_{BML}^s(x, y) \propto P_{BML}^{s-1}(x, y) \prod_{ij \in K} \frac{[n_{ij}^{s-1} + \alpha_{ij}(x, y)]^{\mu_{ij}^s} [s-1 - n_{ij}^{s-1} + \beta_{ij}(x, y)]^{1-\mu_{ij}^s}}{\alpha_{ij}(x, y) + \beta_{ij}(x, y) + s-1}, \quad (x, y) \in A. \quad (7)$$

A Bayesian sequence of predictions on where the mobile unit 20 is located follows from functions (6) or (7) by using the mean or mode of the posterior distribution obtained at each measurement epoch. When function (7) is used, the sequence of prediction involves two distinct prior distributions, beta and discrete uniform, and the methodology is referred to as doubly-Bayesian. This completes the summary of Appendix A.

A description of the mobile unit 20 location prediction process depicted in Figure 4 will now be given. Considered in light of the accompanying appendices A and B and referring to Figures 4A-4E, the location prediction process in accordance with the subject invention, as noted above, is implemented in software which resides in the computer apparatus 23 located at the geolocation server 22 (Figure 3).

The process, referred to hereinafter as the geolocation process, begins at step 30 (Figure 4A) where it is waiting for a new location request to arrive. The geolocation process continually checks for the arrival of a location request at step 32, and if no such request has arrived, it goes back to the waiting state (step 30). When a location service request arrives, the geolocation process identifies the domain of support, i.e., the set of feasible locations, for the mobile's location based on either the identity of the primary base station of the mobile unit or the identity of the strongest pilot signal reported by the mobile unit. This is indicated in step 34. Pilot signals will hereinafter be referred to simply as "pilots."

In the example shown in Figures 1 and 2, if the primary base station for the mobile unit is base station 14<sub>2</sub>, the region designated 17 in Figures 1 and 2 would be selected as the domain of support for the mobile's location and will be hereinafter referred to as the region *A*. Along with the region *A*, the geolocation process at step 34 also identifies the set of all pilots *K* which are likely to be visible at some of the gridpoints 18 in the region *A*. Next, in step 36 approximations for the conditional probability, conditioned on the mobile unit being at a location  $(x, y) \in A$ , that each pilot in *K* is visible to the mobile are computed using equation (B9) from Appendix B. At this point, two decisions external to the geolocation process must be made. The first decision (step 38) that must be made is which of the ML estimation or sequential Bayes estimation methods should be used. For each estimation procedure (ML or sequential Bayes) the second decision to be made is which type of likelihood, frequentist or Bayes-modified, should be used (steps 40 and 42). These two

decisions result in the four paths marked (2)-(5). Figures 4B-4E correspond to each of the four paths, only one of which would typically be used in any implementation of the geolocation process.

First, suppose path (2) of Figure 4A is chosen which represents the combination of the sequential Bayes estimation method and the frequentist likelihood function. This is shown in Figure 4B. There step 44 assigns a discrete uniform prior probability to all grid points 18 in the set  $A$ . The discrete uniform prior reflects the initial state of no information about the mobile unit's whereabouts, other than the fact that it resides in the region  $A$ . The frequentist likelihood function is initialized to unity and time is set to  $s=1$ . Step 46 then evaluates the frequentist likelihood based on the first set ( $s=1$ ) of visibility measurements using expression (1) noted above. In step 48, the posterior distribution based on the first set of visibility measurements is evaluated using function (6). Expression (6) results in values that must be normalized so that when they are summed over all  $(x,y) \in A$ , the result will be unity. The posterior distribution gives the updated probability distribution for the location of the mobile unit 20. A prediction of the location for the mobile unit 20 is next obtained in step 50 by computing either the mean or mode of the posterior distribution obtained from step 48. The prediction obtained in step 50 corresponds to the first set of measurements ( $s=1$ ). If no further measurements are expected, the geolocation process terminates, otherwise as shown in step 52 it proceeds to a waiting state (step 54) and stays there (via step 56) until another set of measurements is received. At that point, the geolocation process proceeds to step 58 and increments time up to  $s+1$  before proceeding back to step 46 and looping once again through step 48 and step 50 which lead to an updated prediction for the location of the mobile unit 20. Eventually, no further measurements will be expected and the geolocation process will terminate at step 52.

Next, suppose path (3) of Figure 4A, which is depicted in Figure 4C, is chosen which represents the combination of the sequential Bayes estimation method and the Bayes-modified likelihood function. Step 60 directs the calculation, via equations (2) and (3), noted above, of the two parameters for the beta prior distribution that is used for the true unknown pilot visibility probabilities. Step 62 assigns a discrete uniform prior probability to all grid points 18 in the set  $A$ . The discrete uniform prior reflects the initial state of no

information about the mobile unit's whereabouts, other than the fact that it resides in the region  $A$ . The Bayes-modified likelihood function is initialized to unity and time is set to  $s=1$ . Step 64 then evaluates the Bayes-modified likelihood based on the first set ( $s=1$ ) of visibility measurements using functional expression (4). In step 66, the posterior distribution based on the first set of visibility measurements is evaluated using equation (7). Equation (7) gives values that must be normalized so that when they are summed over all  $(x,y) \in A$ , the result will be unity. The posterior distribution gives the updated probability distribution for the location of the mobile unit 20. A prediction of the location for the mobile unit 20 is obtained in step 68 by computing either the mean or mode of the posterior distribution obtained from step 66. The prediction obtained in step 68 corresponds to the first set of measurements ( $s=1$ ). If no further measurements are expected, the geolocation process terminates, otherwise as shown in step 70 it proceeds to a waiting state (step 72) and stays there (via step 74) until another set of measurements is received. At that point, the geolocation process proceeds to step 76 and increments time up to  $s+1$  before proceeding back to step 64 and looping once again through step 66 and step 68 which lead to an updated prediction for the location of the mobile unit 20. Eventually, no further measurements will be expected and the geolocation process will terminate at step 70.

Next, suppose path (4) of Figure 4A is chosen which represents the combination of the ML estimation method and the Bayes-modified likelihood function. Path (4) is shown in Figure 4D where step 78 first directs the calculation, via equations (2) and (3), of the two parameters for the beta prior distributions that are used for the true unknown pilot visibility probabilities. Step 80 initializes the Bayes-modified likelihood function to unity and time is set to  $s=1$ . Step 82 then evaluates, for all  $(x,y) \in A$ , the Bayes-modified likelihood based on the first set ( $s=1$ ) of visibility measurements using expression (4). In step 84, the value of  $(x,y) \in A$  which gives the largest likelihood value is selected as the Bayes-modified maximum likelihood estimator of the mobile unit's location. The estimate obtained in step 84 corresponds to the first set of measurements ( $s=1$ ). If no further measurements are expected, the geolocation process terminates, otherwise as shown in step 86 it proceeds to a waiting state (step 88) and stays there (via step 90) until another set of measurements is received. At that point, the geolocation process proceeds to step 92 and increments time up

to  $s+1$  before proceeding back to step 82 and step 84 which lead to an updated prediction for the location of the mobile unit 20. Eventually, no further measurements will be expected and the geolocation process will terminate at step 86.

Finally, suppose path (5) of Figure 4A and further shown in Figure 4E, is chosen which represents the combination of the ML estimation method and the frequentist likelihood function. As shown, step 94 initializes the frequentist likelihood function to unity and time is set to  $s=1$ . Step 96 then evaluates, for all  $(x,y) \in A$ , the frequentist likelihood based on the first set ( $s=1$ ) of visibility measurements using functional expression (1). In step 98, the value of  $(x,y) \in A$  which gives the largest likelihood value is selected as the maximum likelihood estimator of the mobile unit's location. The estimate obtained in step 98 corresponds to the first set of measurements ( $s=1$ ). If no further measurements are expected, the geolocation process terminates, otherwise as shown in step 100 it proceeds to a waiting state (step 102) and stays there (via step 104) until another set of measurements is received. At that point, the geolocation process proceeds to step 106 and increments time up to  $s+1$  before proceeding back to step 96 and step 98 which lead to an updated prediction for the location of the mobile unit 20. Eventually, no further measurements will be expected and the geolocation process will terminate at step 100.

Each of the iterative procedures described above with respect to paths (2)-(5), operates to provide an improved estimate of the location of the mobile unit 20 as more and more sets of pilot measurements are reported. Each of the procedures can be readily extended to include other measured quantities such as phase offsets by including those data in the respective likelihood formulations. Also, when desirable, the method of the present invention can be modified so that the actual signal strength of the visible pilots can be used rather than a binary representation of the fact that the pilots are or are not visible.

The foregoing description of the preferred embodiment has been presented to illustrate the invention without intent to be exhaustive or to limit the invention to the form disclosed. In applying the invention, modification and variations can be made by those skilled in the pertaining art without departing from the scope and spirit of the invention. It is intended that the scope of the invention be defined by the claims appended hereto, and their equivalents.

## Appendix A

## Derivation of the Frequentist and Bayes-modified Likelihood Functions

5 The notation used in this Appendix is the same as what has been introduced in the main portion of this application. In what follows, we make the following assumptions:

(i) The visibility of a particular pilot at  $(x, y) \in A$  at a given time  $t$  is independent of whether the pilot was visible at any time  $s < t$ , and is independent of  $t$  as well.

10 (ii) The visibility of pilot  $i$  at time  $t$  is independent of the visibility of pilot  $j$  ( $j \neq i$ ) at time  $t$ .

(iii) Throughout the duration of the prediction process, the mobile is relatively stationary, i.e., the mobile unit remains in its initial sub-cell.

15 Defining  $L^0(x, y) = 1, \forall (x, y) \in A$ , a recursive form for the exact likelihood function through the first  $s$  measurement epochs for the unknown location of the mobile based on (i)-(iii) is

$$L^s(x, y) \propto L^{s-1}(x, y) \prod_{ij \in K} [\theta_{ij}(x, y)]^{\mu_{ij}^s} [1 - \theta_{ij}(x, y)]^{1 - \mu_{ij}^s}, \quad (x, y) \in A. \quad (A1)$$

The functions  $\theta_{ij}(x, y)$  in (A1) are unknown, so the exact likelihood function is not calculable. Replacing the  $\theta_{ij}(x, y)$  with their approximations  $\tilde{\theta}_{ij}(x, y)$ , defined by equation (B9) in Appendix B, gives the definition of the frequentist likelihood function  $L_{ML}^s(x, y)$  shown in expression (1).

20

The frequentist likelihood function (1) was derived under the assumption that a family of functions  $\tilde{\theta}_{ij}(x, y)$  accurately approximate the unknown functions  $\theta_{ij}(x, y)$ . The approximations  $\tilde{\theta}_{ij}(x, y)$  are based on the RF model described in Appendix B. The RF model is an approximation to a very complex stochastic process. Bayesian methods allow the uncertainty associated with  $\tilde{\theta}_{ij}(x, y)$  to be incorporated into the model framework.

25 Rather than assuming  $\tilde{\theta}_{ij}(x, y) \equiv \theta_{ij}(x, y)$ , we can model the parameters  $\theta_{ij}(x, y)$  with

independent prior distributions that have mean values equal to  $\tilde{\theta}_{ij}(x,y)$ . For each  $(x,y) \in A$ , we use a prior distribution for  $\theta_{ij}(x,y)$  that is a beta distribution with probability density function given by

$$f_{\theta_{ij}(x,y)}(u) = \frac{1}{B[\alpha_{ij}(x,y), \beta_{ij}(x,y)]} u^{\alpha_{ij}(x,y)-1} (1-u)^{\beta_{ij}(x,y)-1}, \quad 0 \leq u \leq 1 \quad (A2)$$

5 where  $B[\cdot, \cdot]$  is the complete beta function. Note that the two parameters,  $\alpha_{ij}(x,y)$  and  $\beta_{ij}(x,y)$ , to be determined in what follows, depend on both the possible location  $(x,y) \in A$  and the pilot  $ij \in K$ . Whereas the frequentist likelihood is defined by implicitly assuming  $\Pr[\theta_{ij}(x,y) = \tilde{\theta}_{ij}(x,y)] = 1$ , the Bayes-modified likelihood is defined by taking the expected value of the exact likelihood (A1) with respect to the beta distributions defined by

10 (A2). The recursive form of the exact likelihood (A1) does not amend itself to evaluating the expected value as easily as the non-recursive form of the exact likelihood which is

$$L^s(x,y) \propto \prod_{ij \in K} [\theta_{ij}(x,y)]^{n_{ij}^s} [1 - \theta_{ij}(x,y)]^{s - n_{ij}^s}, \quad (x,y) \in A \quad (A3)$$

where  $n_{ij}^s$  is the number of times pilot  $ij$  was visible through the first  $s$  measurement epochs.

Evaluating the expected value of (A3), with respect to the beta distributions defined by (A2)

15 gives the Bayes-modified likelihood function

$$\begin{aligned} L_{BML}^s(x,y) &\propto E \left\{ \prod_{ij \in K} [\theta_{ij}(x,y)]^{n_{ij}^s} [1 - \theta_{ij}(x,y)]^{s - n_{ij}^s} \right\} \\ &= \prod_{ij \in K} \frac{B[n_{ij}^s + \alpha_{ij}(x,y), s - n_{ij}^s + \beta_{ij}(x,y)]}{B[\alpha_{ij}(x,y), \beta_{ij}(x,y)]}, \quad (x,y) \in A. \end{aligned} \quad (A4)$$

20

It is easily verified that equation (A4) has the recursive form shown in the functional expression (4).

25 The definition of the Bayes-modified likelihood function will be complete once values for  $\alpha_{ij}(x,y)$  and  $\beta_{ij}(x,y)$  are specified. Determining what values to use for

0396403496560



$\alpha_{ij}(x, y)$  and  $\beta_{ij}(x, y)$  is an example of the classic Bayesian dilemma – how to specify a prior distribution? An underlying principle in specifying a prior distribution is that any available information about the parameter of interest, in this case  $\theta_{ij}(x, y)$ , should be reflected in the prior distribution that is used. In this case, we have an approximation of

5  $\theta_{ij}(x, y)$  from the RF model, namely  $\tilde{\theta}_{ij}(x, y)$ , which we will use as the mean of the prior distribution for  $\theta_{ij}(x, y)$ . Accordingly, we have the following constraint:

$$\frac{\alpha_{ij}(x, y)}{\alpha_{ij}(x, y) + \beta_{ij}(x, y)} = \tilde{\theta}_{ij}(x, y). \quad (A5)$$

Since we have two unknowns,  $\alpha_{ij}(x, y)$  and  $\beta_{ij}(x, y)$ , we need one additional constraint to uniquely determine both  $\alpha_{ij}(x, y)$  and  $\beta_{ij}(x, y)$ . To obtain the second constraint, we

10 impose that the prior be as vague as possible, subject to the first constraint. To implement this second constraint, we will maximize the variance of the prior, subject to the fixed mean value. The second constraint is consistent with the quantity of the prior knowledge we have about  $\theta_{ij}(x, y)$  and except for anticipating a value near  $\tilde{\theta}_{ij}(x, y)$ , we know nothing else about its potential value. We also restrict attention to beta distributions which have

15 bounded density functions, implying  $\alpha_{ij}(x, y) \geq 1$  and  $\beta_{ij}(x, y) \geq 1$ . The restriction to bounded density functions only rules out “U-shaped and “Slide-shaped” beta distributions which would not typically be intuitive representations of the prior information on  $\theta_{ij}(x, y)$ . The problem of selecting  $\alpha_{ij}(x, y)$  and  $\beta_{ij}(x, y)$  has now been sufficiently constrained so that it can be solved by using the following Lemma.

20 *Lemma*

Consider the family of beta distribution with parameters  $a \geq 1$  and  $b \geq 1$ . The values of  $a$  and  $b$  which maximize the variance, subject to the distribution having a given mean value  $\mu$ , are:  $a = 1$  and  $b = (1 - \mu) / \mu$ , if  $\mu \leq 1/2$ ;  $a = \mu / (1 - \mu)$  and  $b = 1$ , if  $\mu > 1/2$ .

*Proof of Lemma*

25 The variance of the beta distribution is given by  $\sigma^2 = ab / [(a + b + 1)(a + b)^2]$ . Since we have  $\mu = a / (a + b)$ , we can write  $b = a(1 - \mu) / \mu$ , and hence  $\sigma^2 = \mu^2(1 - \mu) / (a + \mu)$ . To

maximize the variance, we need to minimize  $a$ , subject to the constraint that both  $a \geq 1$  and  $b \geq 1$ . Clearly, if  $\mu \leq 1/2$  then we can choose  $a = 1$  and  $b = (1 - \mu) / \mu \geq 1$  will be ensured. Likewise, if  $\mu > 1/2$  then the smallest we can choose  $a$  and still have  $b \geq 1$  is  $a = \mu / (1 - \mu) \geq 1$ .

- 5           Applying the Lemma with  $a = \alpha_{ij}(x, y)$ ,  $b = \beta_{ij}(x, y)$  and  $\mu = \tilde{\theta}_{ij}(x, y)$  gives the formulas in equations (2) and (3) for  $\alpha_{ij}(x, y)$  and  $\beta_{ij}(x, y)$ .

663220" 8496360

**Appendix B****Derivation of the Approximations to the Pilot Visibility Probabilities**

In this Appendix, we drive the approximations  $\tilde{\theta}_{ij}(x,y)$  of the pilot visibility probabilities. The notation used in this Appendix is the same as what has been introduced earlier in this specification. Consider a grid point  $(x,y) \in A$ . As in the above referenced related application Serial No. \_\_\_\_\_, entitled, "Geolocation Estimation Method For CDMA Terminals Based On Pilot Strength Measurements", the RF power received by the mobile unit 20 from sector  $j$  of base station  $i$  is modeled by the expression

$$R_{ij}(x,y) = T_{ij} G_{ij}(x,y) L_{ij}(x,y) F_{ij}(x,y) M_{ij}(x,y) \quad (B1)$$

where  $T_{ij}$  is the transmit power associated with the sector,  $G_{ij}(x,y)$  is the antenna gain for the sector along the direction pointing towards the location  $(x,y) \in A$ ,  $L_{ij}(x,y)$  is the distance loss between the base station associated with the sector and the location  $(x,y) \in A$ ,  $F_{ij}(x,y)$  is the shadow fading factor and  $M_{ij}(x,y)$  is the measurement noise factor, all in absolute, not dB, units. The measurement noise factor is meant to include the effects of fast fading (e.g., Rayleigh/Rician) as well as inaccuracies in the measurement process. If  $\gamma$  denotes the fraction of  $T_{ij}$  that is used for the pilot channel, then  $\gamma R_{ij}(x,y)$  is the pilot channel power received by the mobile unit 20 when it is located at  $(x,y) \in A$ .

The model assumes that the distance loss  $L_{ij}(x,y)$  can be expressed as

$$L_{ij}(x,y) = C_p [d_{ij}(x,y)]^{-\alpha} \quad (B2)$$

where  $d_{ij}(x,y)$  is the distance between the base station associated with the sector and the location  $(x,y) \in A$ , and  $C_p$  and  $\alpha$  are constants. Typically,  $C_p$  takes a value in the range  $10^{-15}$  to  $10^{-10}$  and  $\alpha$  is between 3 and 5 when  $d_{ij}(x,y)$  is expressed in miles. The two parameters,  $C_p$  and  $\alpha$ , moreover are environment specific.

The shadow fading factor,  $F_{ij}(x,y)$ , models the impact of terrain and large structures (e.g., buildings) on signal propagation which create deviations around the signal

attenuation predicted by the deterministic path loss factor. We assume that  $F_{ij}(x, y)$  is lognormally distributed and thus can be written as

$$F_{ij}(x, y) = e^{-\frac{1}{2}\sigma_\phi^2} e^{\phi_{ij}(x, y)} \quad (\text{B3})$$

where  $\phi_{ij}(x, y)$  is a zero mean Gaussian random variable with standard deviation  $\sigma_\phi$ .

5        The measurement noise factor,  $M_{ij}(x, y)$  is also assumed to have a lognormal distribution so that we can write

$$M_{ij}(x, y) = e^{-\frac{1}{2}\sigma_\mu^2} e^{\mu_{ij}(x, y)} \quad (\text{B4})$$

where  $\mu_{ij}(x, y)$  is a zero mean Gaussian random variable with standard deviation  $\sigma_\mu$ .

10       Assume now that the pilot strength measurement carried out and reported to the base station by the mobile unit 20 is the  $E_c/I_o$  value of the corresponding pilot channel signal. This value is the ratio of the pilot channel power from the concerned sector received by the mobile unit 20 to the total power received by the mobile unit including thermal noise, and possibly external interference.

15       By letting  $P_{ij}(x, y)$  denote the strength of the pilot channel associated with the sector as measured by the mobile unit 20 located at  $(x, y) \in A$ , it follows that

$$P_{ij}(x, y) = \frac{\gamma R_{ij}(x, y)}{N_0 + \sum_{kl \in K} R_{kl}(x, y)} \quad (\text{B5})$$

where  $N_0$  denotes thermal noise and external interference and (as before)  $kl$  is shorthand notation for the pilot associated with sector  $l$  of base station  $k$ .

For convenience, we define

20       
$$C_{ij}(x, y) = T_{ij} G_{ij}(x, y) L_{ij}(x, y). \quad (\text{B6})$$

Observe that the expected value of  $R_{ij}(x, y)$  is equal to  $C_{ij}(x, y)$ . It is implicitly assumed that the shadow fading and measurement noise factors are uncorrelated. We approximate  $P_{ij}(x, y)$  (B5) by the following expression

6692240" 84965660

$$Z_{ij}(x, y) = \frac{\gamma R_{ij}(x, y)}{N_0 + R_{ij}(x, y) + \sum_{\substack{kl \in K \\ kl \neq ij}} C_{kl}(x, y)} \quad (B7)$$

The difference between (B7) and (B5) is that for  $kl \neq ij$ , the  $R_{kl}(x, y)$  quantities have been replaced by their mean values. Using (B3) and (B4), we can write

$$Z_{ij}(x, y) = \frac{be^{\xi}}{a + ce^{\xi}} \quad (B8)$$

5 where

$$\begin{aligned} a &= e^{\frac{1}{2}(\sigma_{\phi}^2 + \sigma_{\mu}^2)} [N_0 + \sum_{\substack{kl \in K \\ kl \neq ij}} C_{kl}(x, y)] \\ b &= \gamma C_{ij}(x, y) \\ c &= C_{ij}(x, y) \\ \xi &= \phi_{ij}(x, y) + \mu_{ij}(x, y). \end{aligned}$$

The probability that the mobile unit receives pilot  $ij$  with a signal strength in excess of  $T$  is

$$\begin{aligned} \theta_{ij}(x, y) &= \Pr[ P_{ij}(x, y) > T ] \\ &\cong \Pr[ Z_{ij}(x, y) > T ] \\ &\equiv \tilde{\theta}_{ij}(x, y). \end{aligned}$$

It can be easily shown that

10

$$\tilde{\theta}_{ij}(x, y) = Q \left[ -\frac{\ln \left( \frac{b}{a} T^{-1} - \frac{c}{a} \right)}{\sigma} \right] \quad (B9)$$

15 where  $\sigma = (\sigma_{\phi}^2 + \sigma_{\mu}^2)^{1/2}$  and  $Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-\frac{1}{2}t^2} dt$ .

Note that the expression for  $\tilde{\theta}_{ij}(x, y)$  depends on the following unknown parameters:  $\sigma_{\phi}^2$ ,  $\sigma_{\mu}^2$ ,  $N_0$ ,  $C_p$  and  $\alpha$ . As in the above noted related application entitled,

20 "Geolocation Estimation ....", in the present invention, pilot strength measurements are first

[illegible]

Claims

- 1           1. A method of predicting the location of a CDMA mobile communications unit in a  
2 wireless communications service area, comprising the steps of:
- 3           (a) receiving measured attribute information from a mobile unit location in the  
4 service area, said attribute information being specific to the location of the mobile unit in the  
5 service area;
- 6           (b) computing the probability of the mobile unit being at a specific location in the  
7 service area in response to said received attribute information using a likelihood probability  
8 function;
- 9           (c) generating an output indicative of the likelihood of the mobile unit being at said  
10 location in the service area.
- 1           2. A method according to claim 1 wherein said likelihood probability function  
2 comprises an iterative procedure for producing a maximum likelihood estimator of the  
3 mobile unit's location in this service area.
- 1           3. A method according to claim 2 wherein the maximum likelihood estimator  
2 comprises the coordinates in the service area which maximizes the likelihood probability  
3 function.
- 1           4. A method according to claim 2 wherein said likelihood probability function  
2 comprises a frequentist likelihood function.
- 1           5. A method according to claim 2 wherein said likelihood probability function  
2 comprises a Bayes-modified likelihood function.
- 1           6. A method according to claim 1 wherein said procedure comprises a sequential  
2 Bayesian type of procedure.

1           7. A method according to claim 6 wherein said sequential Bayesian type of  
2 procedure uses a frequentist likelihood function.

1           8. A method according to claim 6 wherein said sequential Bayesian type of  
2 procedure uses a Bayes-modified likelihood function.

1           9. A method according to claim 1 and wherein said attribute information comprises  
2 a pilot signal strength measurement of at least one visible pilot signal at said location of the  
3 mobile unit.

1           10. A method according to claim 9 and additionally including a step (d) prior to step  
2 (b) of reducing the effective size of the service area so as to reduce the number of  
3 computations required in step (b).

1           11. A method of claim 9 and following step (a) and before step (b), additionally  
2 including a step (d) of identifying a region A of support for the mobile unit in the service  
3 area and (e) identifying a set of possible pilot signals which can be detected by the mobile  
4 unit in said region of support.

1           12. A method of claim 11 and additionally including a step (f) of computing an  
2 approximation  $\tilde{\theta}_{ij}(x, y)$  of the probability the mobile unit detects a pilot signal at said  
3 specific location (x, y) in said region A for all pilot signals in said set of possible pilot  
4 signals and where j is a sector of a multi-sector base station i in said region of support.

1           13. A method of claim 12 wherein  $\tilde{\theta}_{ij}(x, y)$  is derived from an RF model for the RF  
2 power  $R_{ij}(x, y)$  received by the mobile unit and where the model is of the form of

$$R_{ij}(x, y) = T_{ij} G_{ij}(x, y) L_{ij}(x, y) F_{ij}(x, y) M_{ij}(x, y)$$

669240" 84965660



3 where  $T_{ij}$  is the transmit power associated with the sector j of base station i,  $G_{ij}(x, y)$  is the  
4 antenna gain for the sector j of base station i along the direction pointing towards the  
5 location (x, y) within A,  $L_{ij}(x, y)$  is the distance loss between the base station i associated  
6 with the sector j and the location (x, y) within A,  $F_{ij}(x, y)$  is the shadow fading factor and  
7  $M_y(x, y)$  is the measurement noise factor.

1 14. A method according to claim 13 wherein said attribute information comprises  
2 the measured visibility of a pilot signal transmitted from a base station and the likelihood  
3 probability function comprises an iterative frequentist maximum likelihood (ML) function in  
4 the form of

$$L_{ML}^s(x, y) \propto L_{ML}^{s-1}(x, y) \prod_{ij \in K} [\tilde{\theta}_{ij}(x, y)]^{\mu_{ij}^s} [1 - \tilde{\theta}_{ij}(x, y)]^{1 - \mu_{ij}^s}, \quad (x, y) \in A$$

5  
6  
7  
8  
9 where x and y comprise rectilinear coordinates in the service area A, s is the number  
10 of measurement epochs, and  $\mu_{ij}^s$  is equal to one or zero depending on whether the mobile  
11 unit can detect a pilot signal ij at measurement epoch s.

1 15. A method according to claim 13 wherein said attribute information comprises  
2 the measured visibility of a pilot signal transmitted from a base station and the likelihood  
3 probability function comprises an iterative Bayes-modified maximum likelihood (ML)  
4 estimator in the form of

$$L_{BML}^s(x, y) \propto L_{BML}^{s-1}(x, y) \prod_{ij \in K} \frac{[n_{ij}^{s-1} + \alpha_{ij}(x, y)]^{\mu_{ij}^s} [s - 1 - n_{ij}^{s-1} + \beta_{ij}(x, y)]^{1 - \mu_{ij}^s}}{\alpha_{ij}(x, y) + \beta_{ij}(x, y) + s - 1}, \quad (x, y) \in A$$

5  
6  
7  
8 where x and y comprise rectilinear coordinates in the service area A s is the number of  
9 measurement epochs,  $n_{ij}^{s-1}$  is the number of times a pilot signal in section j of a multi-sector  
10 base station i is visible through the first s-1 measurement epochs,  $\alpha_{ij}(x, y)$  and  $\beta_{ij}(x, y)$  are

11 parameters of a beta distribution, and  $\mu_{ij}^s$  is equal to one or zero depending on whether the  
12 mobile unit can detect a pilot signal  $ij$  at measurement epoch  $s$ .

1 16. A method according to claim 15 where

$$\alpha_{ij}(x, y) = \begin{cases} 1 & , \text{if } \tilde{\theta}_{ij}(x, y) \leq 1/2 \\ \frac{\tilde{\theta}_{ij}(x, y)}{1 - \tilde{\theta}_{ij}(x, y)} & , \text{if } \tilde{\theta}_{ij}(x, y) > 1/2 \end{cases}$$

$$\beta_{ij}(x, y) = \begin{cases} \frac{1 - \tilde{\theta}_{ij}(x, y)}{\tilde{\theta}_{ij}(x, y)} & , \text{if } \tilde{\theta}_{ij}(x, y) \leq 1/2 \\ 1 & , \text{if } \tilde{\theta}_{ij}(x, y) > 1/2 . \end{cases}$$

1 17. A method according to claim 13 wherein the frequentist likelihood function  
2  $L_{ML}^s(x, y)$  is combined with a discrete uniform prior distribution for the location of the  
3 mobile unit of the form

$$P_{ML}^o(x, y) = \frac{1}{\|A\|}, (x, y) \in A$$

6 to generate a sequential Bayesian procedure which provides a posterior distribution for the  
7 location of the mobile unit of the form

$$P_{ML}^s(x, y) \propto P_{ML}^{s-1}(x, y) \prod_{ij \in K} [\tilde{\theta}_{ij}(x, y)]^{\mu_{ij}^s} [1 - \tilde{\theta}_{ij}(x, y)]^{1 - \mu_{ij}^s}, (x, y) \in A.$$

12 where  $\|A\|$  is the number of grid points contained within  $A$ , and where  $x$  and  $y$  comprise  
13 rectilinear coordinates in the service area  $A$ ,  $s$  is the number of measurement epochs, and

14  $\mu_{ij}^s$  is equal to one or zero depending on whether the mobile unit can detect a pilot signal  
15  $ij$  at measurement epoch  $s$ .

1 18. A method according to claim 13 wherein the frequentist likelihood function  
2  $L_{BML}^s(x, y)$  is combined with a discrete uniform prior distribution for the location of the  
3 mobile unit of the form

$$5 \quad P_{BML}^0(x, y) = \frac{1}{\|A\|}, (x, y) \in A$$

6 to generate a sequential Bayesian procedure which provides a posterior distribution for the  
7 location of the mobile unit of the form

$$9 \quad P_{BML}^s(x, y) \propto P_{BML}^{s-1}(x, y) \prod_{ij \in K} \frac{[n_{ij}^{s-1} + \alpha_{ij}(x, y)]^{\mu_{ij}^s} [s - 1 - n_{ij}^{s-1} + \beta_{ij}(x, y)]^{1 - \mu_{ij}^s}}{\alpha_{ij}(x, y) + \beta_{ij}(x, y) + s - 1}, (x, y) \in A.$$

12 where  $\|A\|$  is the number of grid points contained within  $A$ , and where  $x$  and  $y$  comprise  
13 rectilinear coordinates in the service area  $A$ ,  $s$  is the number of measurement epochs,  $n$  is the  
14 number of times a pilot signal in section  $j$  of a multi-sector base station  $i$  is visible through  
15 the first  $s-1$  measurement epochs,  $\alpha_{ij}(x, y)$  and  $\beta_{ij}(x, y)$  are parameters of a beta  
16 distribution, and  $\mu_{ij}^s$  is equal to one or zero depending on whether the mobile unit can detect  
17 a pilot signal  $ij$  at measurement epoch  $s$ .

1 19. A method according to claim 18 where

$$2 \quad \alpha_{ij}(x, y) = \begin{cases} 1 & , \text{if } \tilde{\theta}_{ij}(x, y) \leq 1/2 \\ \tilde{\theta}_{ij}(x, y) & , \text{if } \tilde{\theta}_{ij}(x, y) > 1/2 \\ 1 - \tilde{\theta}_{ij}(x, y) & \end{cases}$$

$$3 \quad \beta_{ij}(x, y) = \begin{cases} 1 - \tilde{\theta}_{ij}(x, y) & , \text{if } \tilde{\theta}_{ij}(x, y) \leq 1/2 \\ \tilde{\theta}_{ij}(x, y) & , \text{if } \tilde{\theta}_{ij}(x, y) > 1/2. \\ 1 & \end{cases}$$

1           20. Apparatus for estimating the location of a mobile communications unit in the  
2 service area of a wireless communications system comprising:

3           a plurality of base stations and at least one switching center providing a common  
4 access to said plurality of base stations;

5           computing apparatus including a memory, electrically connected to at least one of  
6 said base stations or said switching center, for storing a set of parameters of a model of the  
7 wireless communications system including the RF environment and predicted values of  
8 attribute measurement of the service area and wherein the values of the parameters are  
9 adjusted to obtain a substantial match between measured attribute values at a predetermined  
10 number of locations in the service area and the corresponding initial values predicted by the  
11 model;

12           said computing apparatus further including software for calculating, in response to  
13 one or more attribute values being measured and reported by the mobile unit from a specific  
14 location within the service area, a predicted location of the mobile unit within the service  
15 area using a likelihood probability function; and,

16           circuit means coupled to said computing apparatus for generating an output  
17 indicative of the predicted location of the mobile unit.

653240" 84365660

ABSTRACT

66320" CHANGED

The location of a mobile wireless communication unit in the service area of a CDMA communications system is predicted utilizing two likelihood functions that define maximum likelihood estimators of the mobile unit's location, based on attribute measurements, such as but not limited to pilot signal strength, being made at the location of the mobile unit and reported back to a base station. One of the likelihood functions comprises a frequentist likelihood function and the other comprises a Bayesian-modified likelihood function. The likelihood functions are based on the assumption that there is an RF model which provides the probability a mobile unit is able to detect one or more attributes associated with an arbitrary base station, given it is located at an arbitrary location within the service area. Each of the likelihoods are also incorporated into a sequential Bayesian procedure which outputs a posterior distribution indicative of the location of the mobile unit.

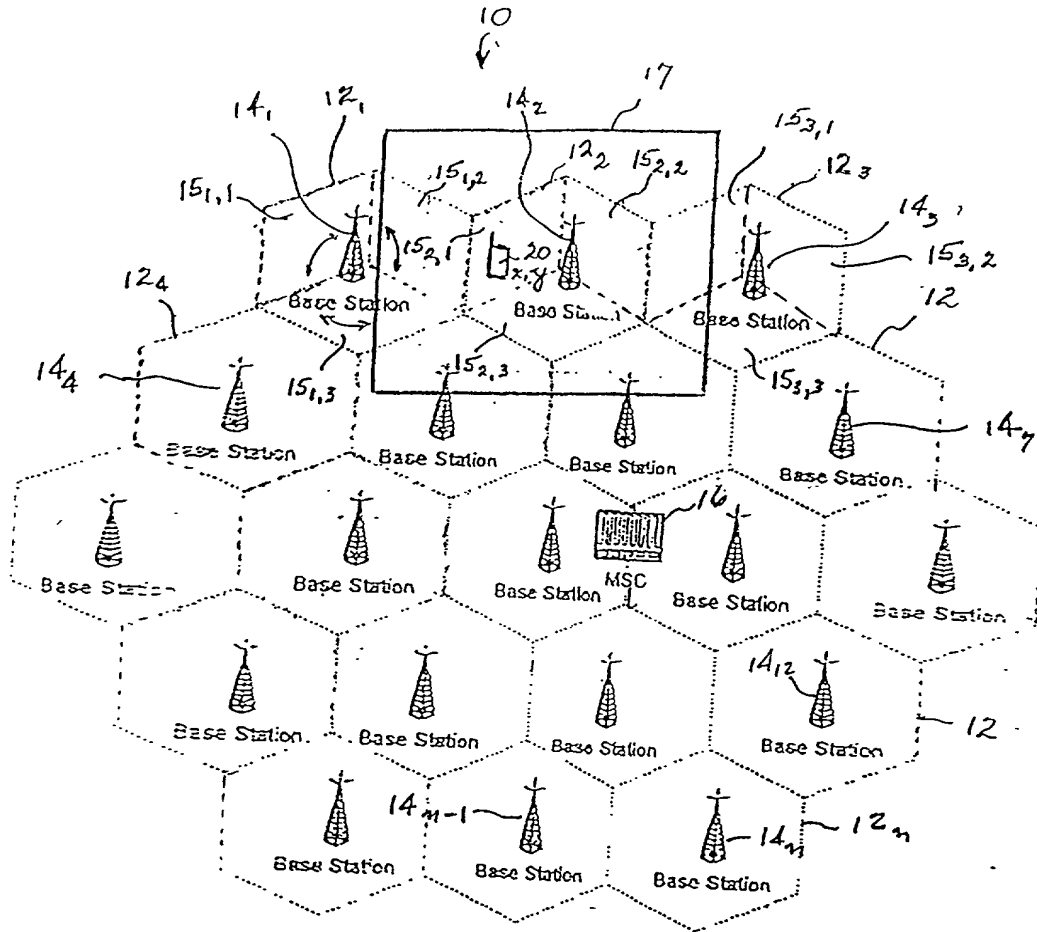
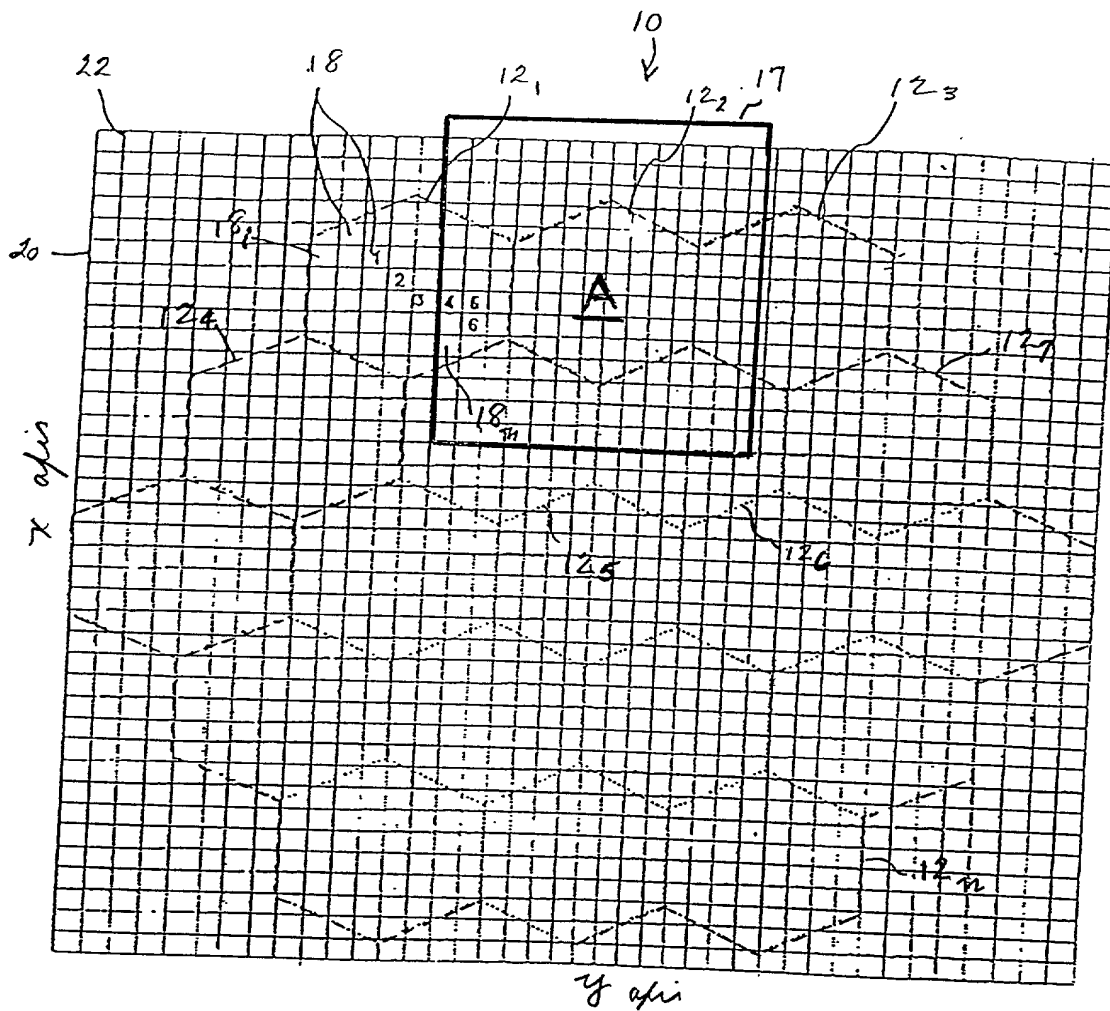


FIG. 1

659270" 84965660



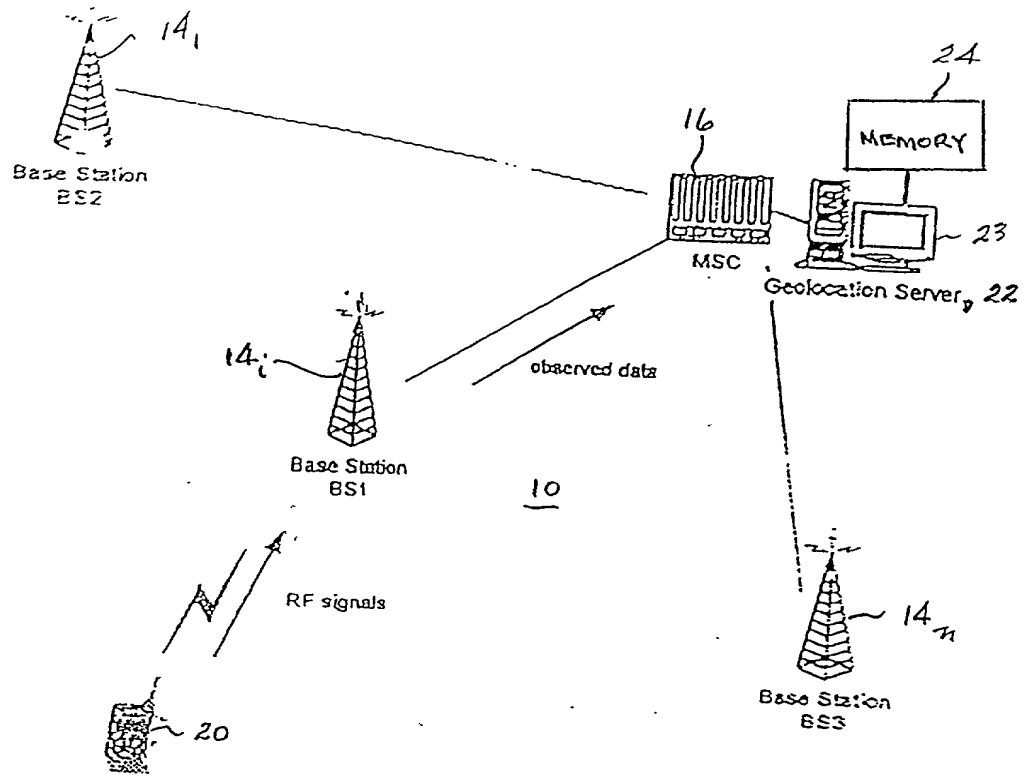


FIG. 3

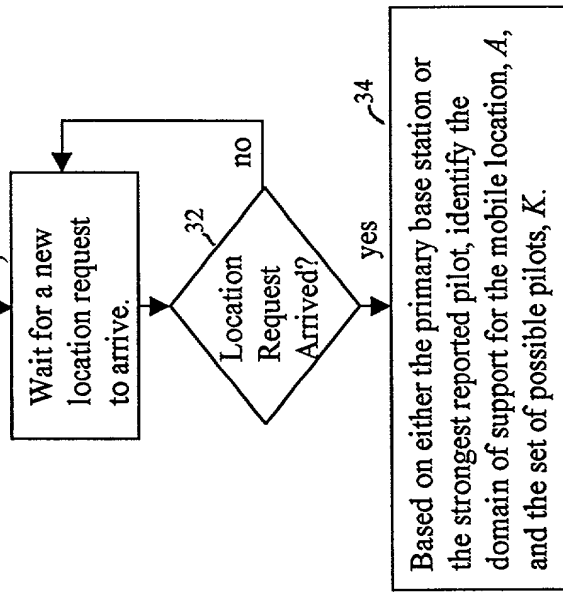
653240 3496360



FIG. 4A	FIG. 4B	FIG. 4E
FIG. 4C	FIG. 4D	

FIG. 4

FIG. 4A



36  
Compute  $\tilde{\theta}_{ij}(x, y)$  for all  $(x, y)$  in  $A$  and for all pilots  $ij$  in  $K$  using equation (B9)

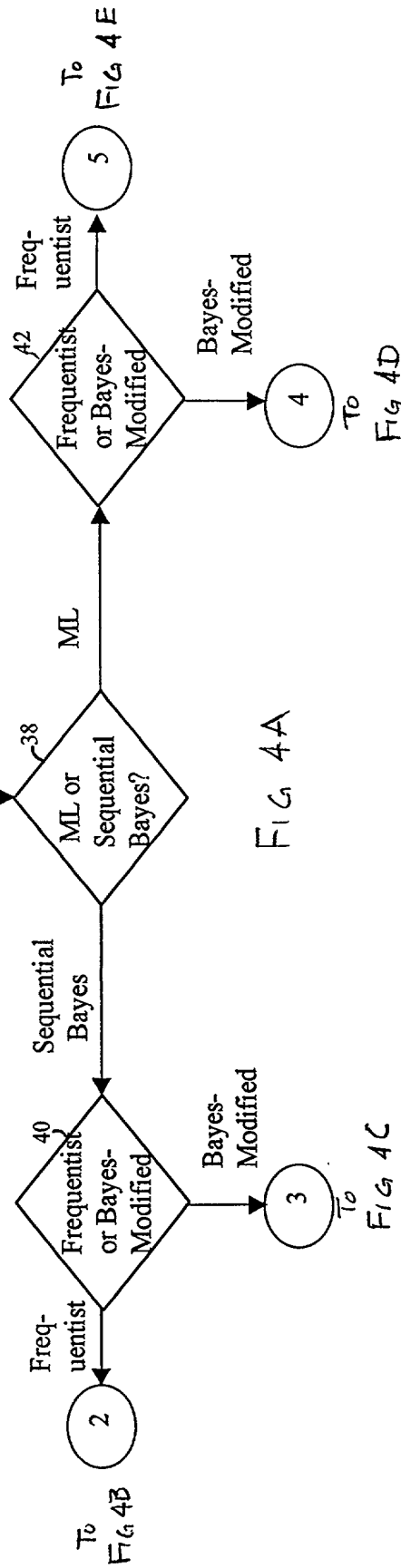


FIG. 4A

From  
FIG. 4A

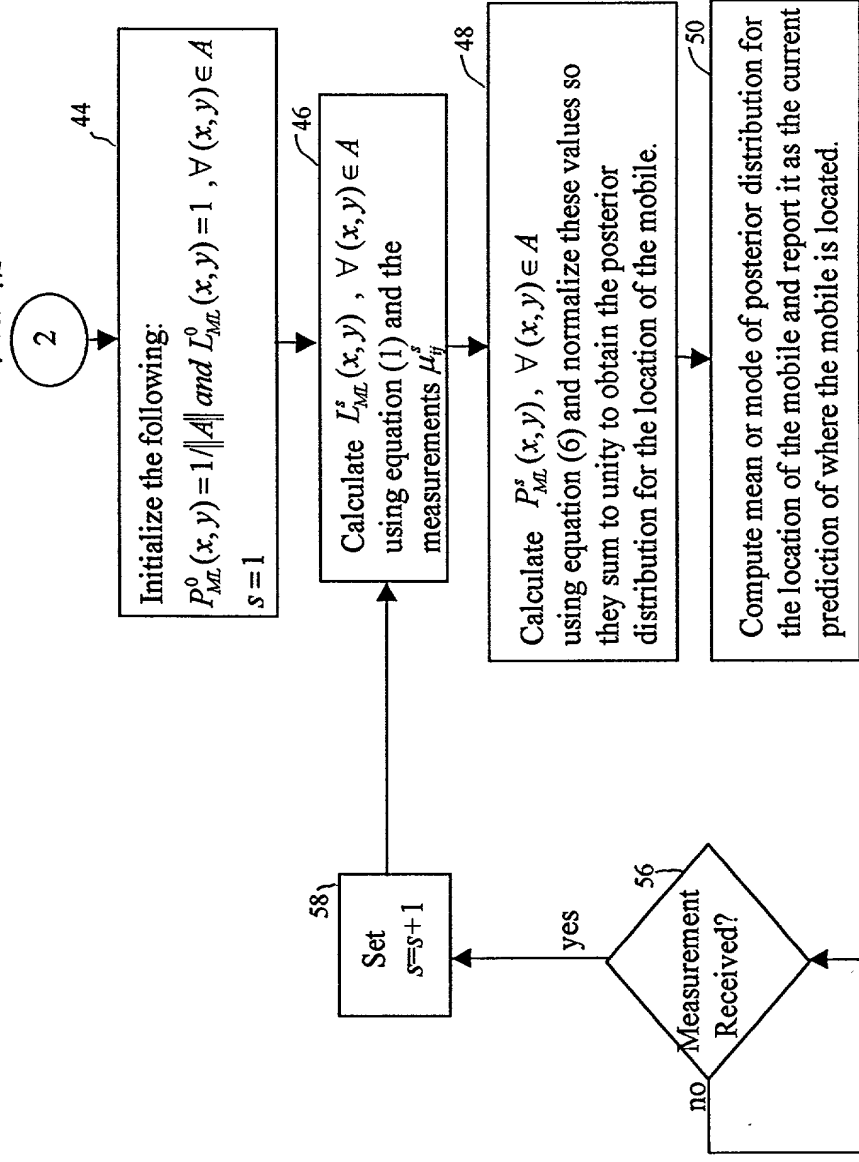


FIG. 4B

FIG. 4A

6/8

FIG. 4A

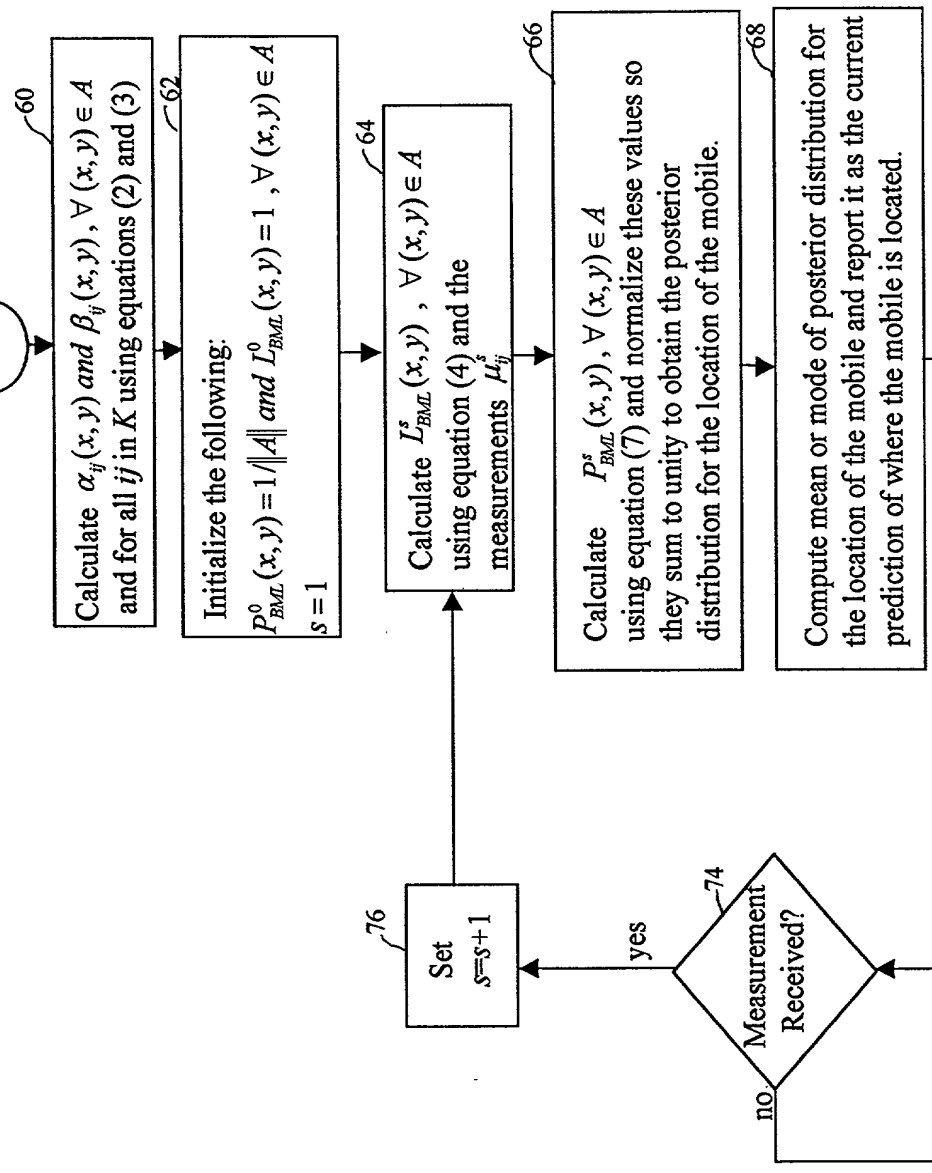


FIG. 4C

FIG. 4A

7/8

FIG. 4A

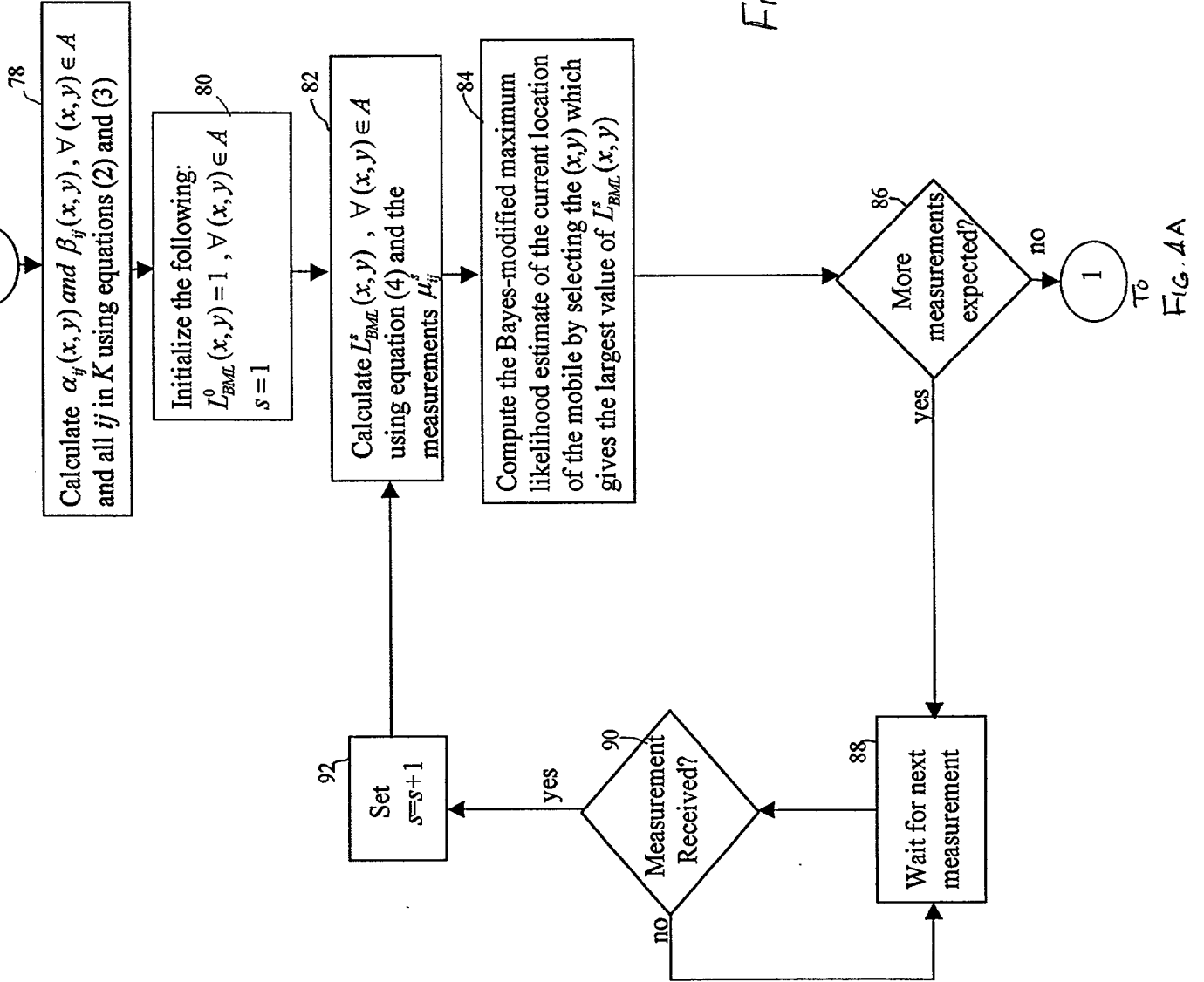


FIG 4D

FIG. 4A

669220" 8496560

From  
FIG. 4A

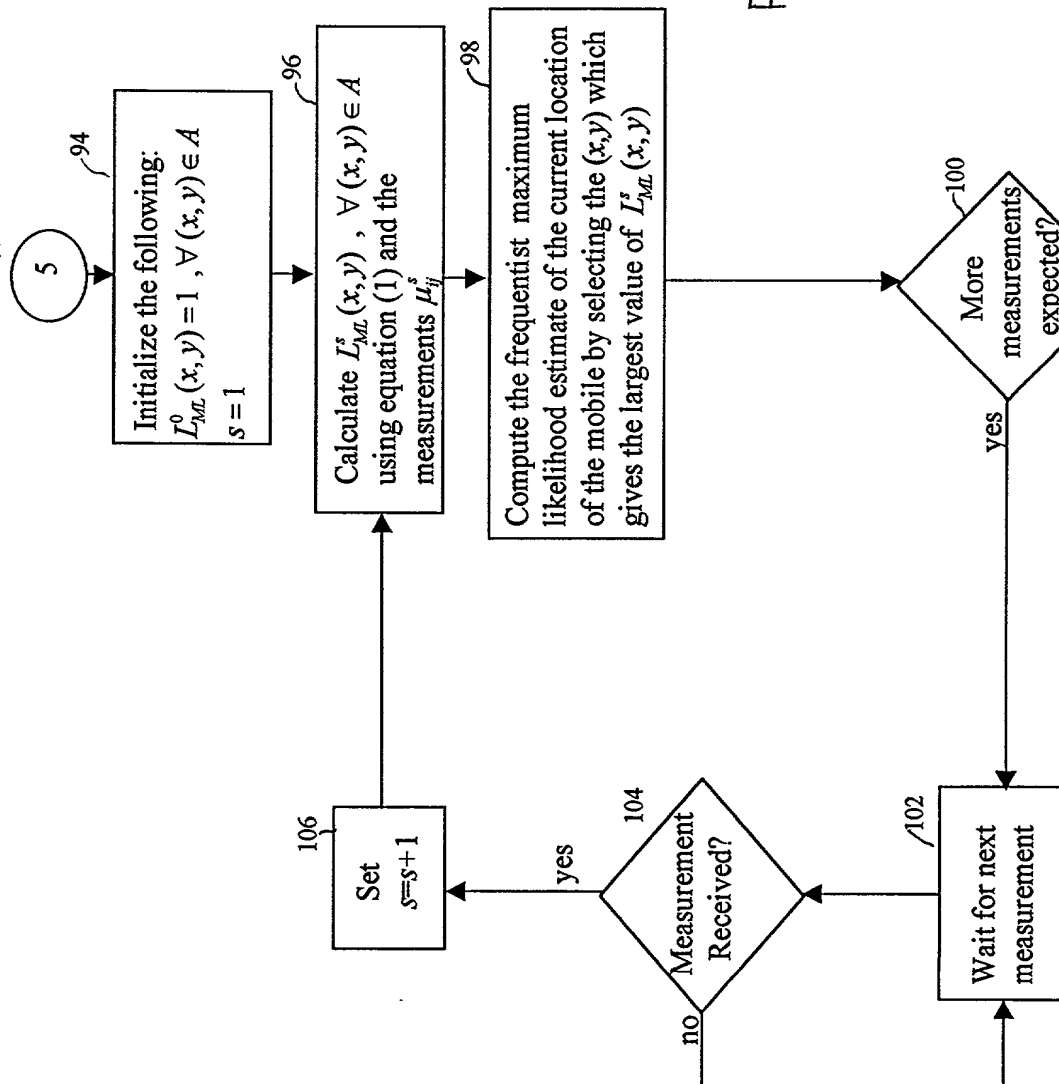


FIG. 4E

To  
FIG. 4A

IN THE UNITED STATES  
PATENT AND TRADEMARK OFFICE

Declaration and Power of Attorney

As the below named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name.

I/We believe I/We am/are the original, inventor(s) of the subject matter which is claimed and for which a patent is sought on the invention entitled **LIKELIHOOD-BASED GEOLOCATION PREDICTION ALGORITHMS FOR CDMA SYSTEMS USING PILOT STRENGTH MEASUREMENTS**, the specification of which *is attached hereto*.

I hereby state that I have reviewed and understand the contents of the above identified specification, including the claims, as amended by an amendment, if any, specifically referred to in this oath or declaration.

I acknowledge the duty to disclose all information known to me which is material to patentability as defined in Title 37, Code of Federal Regulations, 1.56.

I hereby claim foreign priority benefits under Title 35, United States Code, 119 of any foreign application(s) for patent or inventor's certificate listed below and have also identified below any foreign application for patent or inventor's certificate having a filing date before that of the application on which priority is claimed:

None

I hereby claim the benefit under Title 35, United States Code, 120 of any United States application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, 112, I acknowledge the duty to disclose all information known to me to be material to patentability as defined in Title 37, Code of Federal Regulations, 1.56 which became available between the filing date of the prior application and the national or PCT international filing date of this application:

None

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

035648-072699  
669320-8496550

I hereby appoint the following attorney(s) with full power of substitution and revocation, to prosecute said application, to make alterations and amendments therein, to receive the patent, and to transact all business in the Patent and Trademark Office connected therewith:

Lester H. Birnbaum	(Reg. No. 25830)
Richard J. Botos	(Reg. No. 32016)
Jeffery J. Brosemer	(Reg. No. 36096)
Kenneth M. Brown	(Reg. No. 37590)
Donald P. Dinella	(Reg. No. 39961)
Guy Eriksen	(Reg. No. P-41736)
Martin I. Finston	(Reg. No. 31613)
James H. Fox	(Reg. No. 29379)
William S. Francos	(Reg. No. 38456)
Barry H. Freedman	(Reg. No. 26166)
Julio A. Garceran	(Reg. No. 37138)
Mony R. Ghose	(Reg. No. 38159)
Jimmy Goo	(Reg. No. 36528)
Anthony Grillo	(Reg. No. 36535)
Stephen M. Gurey	(Reg. No. 27336)
John M. Harman	(Reg. No. 38173)
Donald E. Hayes, Jr.	(Reg. No. 33245)
John W. Hayes	(Reg. No. 33900)
Michael B. Johannesen	(Reg. No. 35557)
Mark A. Kurisko	(Reg. No. 38944)
Irena Lager	(Reg. No. 39260)
Christopher N. Malvone	(Reg. No. 34866)
Scott W. McLellan	(Reg. No. 30776)
Martin G. Meder	(Reg. No. 34674)
Geraldine Monteleone	(Reg. No. 40097)
John C. Moran	(Reg. No. 30782)
Michael A. Morra	(Reg. No. 28975)
Gregory J. Murgia	(Reg. No. 41209)
Claude R. Narcisse	(Reg. No. 38979)
Joseph J. Opalach	(Reg. No. 36229)
Neil R. Ormos	(Reg. No. 35309)
Eugen E. Pacher	(Reg. No. 29964)
Jack R. Penrod	(Reg. No. 31864)
Daniel J. Piotrowski	(Reg. No. P-42079)
Gregory C. Ranieri	(Reg. No. 29695)
Scott J. Rittman	(Reg. No. 39010)
Eugene J. Rosenthal	(Reg. No. 36658)
Bruce S. Schneider	(Reg. No. 27949)
Ronald D. Slusky	(Reg. No. 26585)
David L. Smith	(Reg. No. 30592)
Patricia A. Verlangieri	(Reg. No. P-42201)

669320-0196550

John P. Veschi	(Reg. No. 39058)
David Volejnicek	(Reg. No. 29355)
Charles L. Warren	(Reg. No. 27407)
Eli Weiss	(Reg. No. 17765)

I hereby appoint the attorney(s) on ATTACHMENT A as associate attorney(s) in the aforementioned application, with full power solely to prosecute said application, to make alterations and amendments therein, to receive the patent, and to transact all business in the Patent and Trademark Office connected with the prosecution of said application. No other powers are granted to such associate attorney(s) and such associate attorney(s) are specifically denied any power of substitution or revocation.

Full name of 1st joint inventor: Kirk K. CHANG

Inventor's signature

*Kirk K. Chang*

Date

*7/7/1999*

Residence: Morganville, Monmouth, New Jersey

Citizenship: Taiwan

Post Office Address: 6 Regina Road, Morganville, New Jersey 07751

6593240-24365E50



Inventor's signature Daniel Robert Ashe Date 7/7/99

Citizenship: USA

Post Office Address: 27 Victoria Drive, Eatontown, New Jersey 07724

Full name of 3rd joint inventor: Kiran M. REGE

Inventor's signature  Date 7/7/1999


Residence: Marlboro, Monmouth, New Jersey

Citizenship: India

Post Office Address: 1 Hull Court, Marlboro, New Jersey 07746

663320" 84353550

Full name of 4th joint inventor: Yung-Terng WANG

Inventor's signature  Date 7/16/99

Residence: : Marlboro, Monmouth, New Jersey

Citizenship: USA

Post Office Address: #7 Peach Tree Ct., Marlboro, New Jersey 07746

060320"0705550

**ATTACHMENT A**

Attorney Name(s):	<u>Raymond C. Stewart</u>	Reg. No.:	<u>21,066</u>
	<u>Joseph A. Kolasch</u>		<u>22,463</u>
	<u>James M. Slattery</u>		<u>28,380</u>
	<u>Donald J. Daley</u>		<u>34,313</u>
	<u>John A. Castellano</u>		<u>35,094</u>

Telephone calls should be made to Birch Stewart Kolasch & Birch, LLP at:

Phone No.: (703) 205-8000

Fax No.: (703) 205-8050

All written communications are to be addressed to:

**BIRCH, STEWART, KOLASCH & BIRCH, LLP,**  
P.O. BOX 747  
Falls Church, Virginia 22040-0747

1563220-2436550